

**Iterative System for Universal Functional  
Optimization (UFO) - Version 1998**

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Technical report No. 766  
January 1999

**Abstract**

This report contains a description of the iterative system for universal functional optimization UFO, version 1998. Demo-version which work up to 19 variables can easily be obtained either by using the anonymous ftp (address <ftp://ftp.uivt.cas.cz/pub/msdos/ufo>), files `ufopcd.arj`, `ufohpd_tar.gz`, `ufosund_tar.gz`, `ufosgid_tar.gz`, or from the web homepage <http://www.uivt.cas.cz/luksansoftware>.

**Keywords**

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Subroutines for Testing Large Sparse and  
Partially Separable Unconstrained and Equality  
Constrained Optimization Problems

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**Abstract**

This report contains a description which can be used for testing large-scale optimization codes. These subroutines can easily be obtained either by using the anonymous ftp (address <ftp://ftp.uivt.cas.cz/pub/msdos/opt>, files TEST14.FOR, TEST15.FOR, TEST20.FOR) or from the web homepage <http://www.uivt.cas.cz/luksansoftware>. Furthermore, all test problem contained in these subroutines are presented in the analytic form.

**Keywords**

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# A Continuous-Time Hopfield Net Simulation of Discrete Neural Networks

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## Abstract

We investigate the computational power of continuous-time symmetric Hopfield nets. Since the dynamics of such networks are governed by Liapunov (energy) functions, they cannot generate infinite nondamping oscillations, and hence cannot simulate arbitrary (potentially divergent) discrete computations. Nevertheless, we prove that any *convergent* fully parallel computation by a network of  $n$  discrete-time binary neurons, with in general asymmetric interconnections, can be simulated by a symmetric continuous-time Hopfield net containing  $14n + 6$  units using the saturated-linear sigmoid activation function. In terms of standard discrete computation models this result implies that any polynomially space-bounded Turing machine can be simulated by a polynomially size-increasing sequence of continuous-time Hopfield nets. Similar techniques as here yield corresponding results on the convergence time and computational power of discrete-time Hopfield nets.

## Keywords

neural networks, continuous-time Hopfield nets, analog computation, computational power

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Globally convergent variable metric method for  
nonconvex nondifferentiable unconstrained  
minimization<sup>1</sup>

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Technical report No. 775  
April 1999

**Abstract**

A special variable metric method is given for finding stationary points of locally Lipschitz continuous functions which are not necessarily convex or differentiable. Time consuming quadratic programming subproblems do not need to be solved. Global convergence of the method is established. Some encouraging numerical experience is reported.

**Keywords**

Nonsmooth minimization, nonconvex minimization, numerical methods, variable metric methods, global convergence

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# Geodynamic Analyses of the Himalayas and the Andaman Islands Based on Mathematical Simulations of Geodynamics Processes<sup>1</sup>

Jiří Nedoma

Technical report No. 776  
1999

### Abstract

The origin of the Himalayas, the highest mountain chain in the world, has been the most interesting problem from the geodynamic point of view. Plate tectonics, which considers the lithosphere as a number of rigid plates which are in a contact along their boundaries, can explain many of the features of the Indian subcontinent. Molnar and Tapponier (1975) and then many authors consider most of the large scale tectonics of Asia to be a result of the India-Eurasia collision. The collision of Indian plate and Eurasian plate involves the large scale of active continental deformation and therefore a rather diffuse seismicity prevails in this region. Moreover, the volcanism in its separate parts is indicated. To analyze these regions from the geodynamic point of view we shall construct the geodynamic models of these regions in the form of vertical profiles. For this reason the map of the lower boundary of the Indian lithospheric plate as well as new geodynamic models of the Himalayas and the subduction of Indian plate below the Burmese plate will be given and discussed. In the paper the stress-strain analyzes across the Nanda Devi profile as well as the profile across the Andaman Islands will be numerically computed and then the obtained results will be analysed. While geologist presume the contact boundary between the Indian plate and the Eurasian plate to be the Indus Tsangpo Thrust, the obtained results of the numerical computations indicate that the contact between the Indian lithospheric plate and the Eurasian plate is the Main Boundary Thrust and the Main Central Thrust is the deep fault from the Himalayan evolution period before 40 Ma.

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<sup>1</sup>Presented in a short version as "Geodynamic Analysis of the Himalayas and the Andaman Islands" on the 29th General Assembly of the IASPEI, August 18-28, 1997, Thessaloniki

## **Keywords**

Geodynamics, mathematical simulation of geodynamic processes, Himalayas, Andaman Islands

**Probabilistic Analysis of Dempster–Shafer  
Theory  
Part Three**

Ivan Kramosil<sup>1</sup>

Technical report No. 777  
April, 1999

**Abstract**

Dempster–Shafer theory is an interesting and useful mathematical tool for uncertainty quantification and processing. From one point of view it can be seen as an alternative apparatus to probability theory and mathematical statistics based on this probability calculus, as D.–S. theory can be developed in a way quite independent of probability theory, beginning with a collection of more or less intuitive demands which an uncertainty degree calculus should meet. On the other side, however, D.–S. theory can be developed also as a particular sophisticated application of probability theory, using the notion of non–numerical, in particular, set–valued random variables (random sets) and their numerical characteristics. This later aspect enables to generalize D.–S. theory beyond its classical scopes using appropriately the apparatus of probability theory and measure theory.

This report is the third part of a surveyal work cumulating, and presenting in a systematic way, some former author’s ideas and achievements dealing with applications of probability theory and mathematical statistics when defining, developing, and generalizing various parts of D.–S. theory. The more detailed contents of this report can be understood from the list of the titles of the particular chapters presented just below.

**Keywords**

Dempster–Shafer theory, probability theory, belief function, random variable, random set

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# Conjugate Gradient Methods for Saddle Point Systems<sup>1</sup>

Ladislav Lukšan      Jan Vlček

Technical report No. 778  
September 1999

## **Abstract**

Conjugate gradient methods have proved to be very efficient for solving saddle point systems, especially with indefinite preconditioning. These systems are indefinite and have a  $2 \times 2$  block structure. We study various structured conjugate gradient methods and prove their theoretical properties.

## **Keywords**

Nonlinear programming, sparse problems, equality constraints, saddle point systems, indefinite systems, indefinite preconditioners, conjugate gradient methods.

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Numerical Experience with Iterative Methods  
for Equality Constrained Nonlinear  
Programming Problems<sup>1</sup>

Ladislav Lukšan, Jan Vlček

Technical report No. 779

**Abstract**

We show that indefinitely preconditioned symmetric Krylov-subspace methods are very efficient for solving linearized KKT systems arising in equality constrained optimization. We give a numerical comparison of various Krylov subspace methods in three different forms (original system, null space transformation, range space transformation) and two basic approaches (line search, trust region). Furthermore, we give a survey of our previous results concerning indefinite preconditioners and prove new propositions.

**Keywords**

Nonlinear programming, sparse problems, equality constraints, inexact Newton method, line search methods, trust region methods, indefinite systems, indefinite preconditioners, symmetric Krylov subspace methods, residual smoothing, computational experiments.

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<sup>1</sup>This work was supported by grant GA ČR 201/00/0080 and by MURST 1999 Programma di Cofinanziamento, Numerical Methods for Evolutionary Problems

**VARIABLE METRIC METHODS FOR  
UNCONSTRAINED OPTIMIZATION<sup>1</sup>**

Ladislav Lukšan<sup>2</sup>      Emilio Spedicato<sup>3</sup>      Jan Vlček<sup>4</sup>

Technical report No. 780  
May 1999

**Abstract**

Variable metric or quasi-Newton methods are well known and commonly used in connection with unconstrained optimization, since they have good theoretical and practical convergence properties. Although these methods were originally developed for medium-size dense problems, their modifications based either on sparse, partitioned or limited-storage updates are very efficient large-scale sparse problems. Very significant applications of these methods also appear in nonlinear least-squares approximation and nonsmooth optimization. In this contribution, we give an extensive review of variable metric methods and their use in various optimization fields.

**Keywords**

Quasi-Newton methods, variable metric methods, unconstrained optimization, nonlinear least-squares, nonsmooth optimization, sparse problems, limited storage methods.

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## Some Afterthoughts on Hopfield Networks

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Technical report No. 781  
May 1999

### Abstract

The present paper investigates four relatively independent issues, each in one section, which complete our knowledge regarding the computational aspects of popular Hopfield nets [?]. In Section ??, the computational equivalence of convergent asymmetric and Hopfield nets is proved with respect to the network size. In Section ??, the convergence time of Hopfield nets is analyzed in terms of bit representations. In Section ??, a polynomial time approximate algorithm for the minimum energy problem is shown. In Section ??, the Turing universality of analog Hopfield nets is studied.

### Keywords

Hopfield networks, computational power, convergence time, minimum energy problem, analog networks

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The Computational Limits  
to the Cognitive Power of the Neuroidal Tabula  
Rasa

Jiří Wiedermann <sup>1</sup>

Technical report No. 786  
July, 1998

**Abstract**

The neuroidal tabula rasa (NTR) as a hypothetical device which is capable of performing tasks related to cognitive processes in the brain was introduced by L.G. Valiant in 1994. Neuroidal nets represent a computational model of the NTR. Their basic computational element is a kind of a programmable neuron called neuroid. Essentially it is a combination of a standard threshold element with a mechanism that allows modification of the neuroid's computational behaviour. This is done by changing its state and the settings of its weights and of threshold in the course of computation. The computational power of an NTR crucially depends both on the functional properties of the underlying update mechanism that allows changing of neuroidal parameters and on the universe of allowable weights. We will define instances of neuroids for which the computational power of the respective finite-size NTR ranges from that of finite automata, through Turing machines, upto that of a certain restricted type of BSS machines that possess super-Turing computational power. The latter two results are surprising since similar results were known to hold only for certain kinds of analog neural networks.

**Keywords**

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## Upper bounds for gentle branching programs

Stanislav Žák<sup>1</sup>

Technical report No. 788

November 17, 1999

### **Abstract**

In the theory of branching programs there are many results giving lower bounds for restricted branching programs. In [?] (and also in [?]) - among other results - it is demonstrated that the functions used for proving lower bounds are in fact easy functions in many cases. "Easy" means that the functions in question are computable on polynomially sized so-called gentle branching programs ("gentle" is a restriction based on a more careful observation of the nature of computations).

The present report continues this trend and proves that also the functions used for proving the latest lower bounds (Beame et al. [?], Ajtai [?]) are easy in the sense above. Some other results concerning upper bounds are added.

### **Keywords**

branching programs, upper bounds

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Dimension-independent approximation by  
neural networks and its comparison with linear  
approximation

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Technical report No. 789  
October 1999

**Abstract**

In this paper, we compare rates of approximation achievable using any linear approximation method with rates of approximation by neural networks. We compare such rates in terms of the worst-case errors in approximation by  $n$ -dimensional linear subspaces and neural networks with  $n$  hidden units in the last hidden layer. We develop a general framework for such comparisons for sets of multivariate functions either computable by various types of network computational units or approximable with dimension-independent rates by networks with such units. Applying this approach to perceptron networks, we derive lower bounds on the worst-case errors in linear approximation of sets of functions computable by perceptrons with periodic and sigmoidal activation functions.

**Keywords**

linear and nonlinear approximation, Kolmogorov width, dimension-independent approximation, curse of dimensionality, variation with respect to a set of functions, complexity of neural networks, one-hidden-layer perceptron networks

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Intelligence as Large–Scale Computational  
Learning Phenomenon  
(A Short Version)

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Technical report No. 792  
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**Abstract**

A coherent, machine independent computational theory of mind is proposed. The mind is presented as a specific learning algorithm whose functionality is inspired by the cognitive abilities of real brains. The respective interactive self–organizing learning mechanism, formally represented by the so–called *cogitoid*, has been recently introduced by the author.

It is shown that under a suitable training a cogitoid is able to acquire, besides abilities to handle the basic cognitive tasks, also a behavior that in its manifestation resembles that of the mind. The respective development will subsequently feature the emergence of more complex patterns of behaviour, of the self concept, and of elements of abstract thinking along with the rudimentary consciousness. Eventually, language understanding, acquisition and generation will become possible, accompanied by fully developed consciousness enabling further development of abstract thinking.

A thought experiment will show that in principle a properly trained cogitoid could pass the Turing test.

At the conclusion of the paper the implications of the previous results to various issues addressed in the philosophy of mind, such as the problem of phenomenology, computational limits to the mind’s cognitive power, its efficient implementation, or its modularity are discussed. In this light also some aspects of Searle’s Chinese room experiment are addressed. Finally, the problem of the future of the Turing’s tests is tackled. A possible general algorithmic approach to intelligence testing is outlined without restricting it merely to human features.

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## **Keywords**

cognitive computing, interactive computational learning, self-organization, cogitoid, mind evolution and functioning, consciousness, interactive Turing machine, Turing test

# Combinatorial Methods in Analysis<sup>1</sup>

Vlastimil Pták

Technical report No. 793

April 1999

## **Abstract**

A combinatorial approach developed by the author in 1959 is used to explain the connection between limit process and combinatorial properties of families of finite sets.

## **Keywords**

Topological, space, uniform convergence, weak compactness, duality, convex mean, optimization problem, Banach space, number theory

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### Best approximation by linear combinations of characteristic functions of half-spaces

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Věra Kůrková<sup>2</sup>

Andrew Vogt<sup>3</sup>

Technical report No. 795

Nov. 25, 1999

#### Abstract

It is shown that in  $\mathcal{L}_p$ -spaces with  $p \in [1, \infty)$  there exists a best approximation mapping to the set of functions computable by Heaviside perceptron networks with  $n$  hidden units; however for  $p \in (1, \infty)$  such best approximation is not unique and cannot be continuous.

#### Keywords

One-hidden-layer networks, Heaviside perceptrons, best approximation, metric projection, continuous selection, approximatively compact.

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